2016

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the moment of inertia of a circular ring of mass M and radius a about an axis through its centre and perpendicular to its plane.

(b) Write down the radius of gyration of a spherical shell of mass M and radius a about a diameter.

(c) A particle of mass 4 units is placed at the point $(-1, -1, 1)$. What is the product of inertia of the particle about OX-OY?

(d) State the perpendicular axes theorem on moments of inertia.

(e) A rigid body moves freely in space. What is the degree of freedom of the body?

(f) Define the centre of suspension of a compound pendulum.

(g) Define conservative system.

2. Answer the following questions : $2 \times 4 = 8$

(a) A rigid body consists of 3 particles of masses 2, 1, 4 located at $(1, -1, 1)$, $(2, 0, 2)$ and $(-1, 1, 0)$ respectively. Find the moments of inertia about the x, y and z axes.

(b) A rigid body of mass 2 units rotates with angular velocity $\vec{\omega} = (1, 1, -1)$ and has the angular momentum $\vec{\Omega} = (2, 3, -1)$. Find the kinetic energy of the body.

(c) A body with one point fixed rotates with angular velocity $(0, 0, 2)$. Find the magnitude of the velocity of a particle of mass m of the body located at the point $(3, -4, 1)$.

(d) A particle moves under the influence of central force field $f(r)\vec{r}$, where $r = |\vec{r}|$, $\vec{r}$ being the position vector of the particle relative to the centre of force O. Show that the angular momentum of the particle about O is constant.

3. Answer the following questions : 5×3=15

(a) Find the moment of inertia of a uniform triangular lamina about one side.

Or

Find the moment of inertia of a solid sphere of radius a and mass M about a diameter.

(b) State and prove d'Alembert's principle.

Or

Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

(c) Show that a uniform rod of mass m is kinetically equivalent to three particles rigidly connected and situated one at each end of the rod and its middle point, the masses of the particles being $\frac{1}{6}m$, $\frac{1}{6}m$ and $\frac{2}{3}m$.

4. A rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α, show that

$$\omega^2 = \frac{3g}{4a\cos\alpha}$$

Prove also that the direction of reaction at the hinge makes with the vertical an angle

$$\tan^{-1}\left(\frac{3}{4}\tan\alpha\right)$$ 10

Or

(a) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M', starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$$

where a is the length of the plank. 6

(b) Using d'Alembert's principle, derive the general equations of motion of a rigid body. 4

5. *(a)* An elliptical lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation. Prove that the eccentricity is $\frac{1}{2}$. 5

(b) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion. 5

6. Write down the equations of motion of a rigid body in two dimensions under impulsive forces. Two equal uniform rods, *AB* and *AC*, are freely jointed at *A*, and are placed on a smooth table so as to be at right angles. The rod *AC* is struck by a blow at *C* in a direction perpendicular to itself. Show that the resulting velocities of the middle points of *AB* and *AC* are in the ratio 2 : 7. 3+7=10

Or

AB and *CD* are two equal similar rods connected by a string *BC*; *AB*, *BC* and *CD* form three sides of the square. The point *A* of the rod *AB* is struck a blow in a direction perpendicular to the rod. Show that the initial velocity of *A* is seven times that of *D*. 10

★★★