2016

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed : 1×8=8

(a) If A and B are mutually exclusive, what will be the modified statement of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$?

(b) If a non-negative real valued function f is the probability density function of some continuous random variable, then what is the value of $\int_{-\infty}^{\infty} f(x)\,dx$?

(c) What is meant by mathematical expectation of a random variable?

(d) For a Bernoulli random variable X with $P(X=0)=1-p$ and $P(X=1)=p$, write $E(X)$ and $V(X)$ in terms of p.

(e) Can the probabilities of three mutually exclusive events A, B, C as given by $P(A)=\frac{2}{3}$, $P(B)=\frac{1}{4}$ and $P(C)=\frac{1}{6}$ be correct? If not, give reasons.

(f) Test the validity of the following probability distribution :

x	-1	0	1
$p(x)$	0·4	0·4	0·3

(g) Under what condition $\operatorname{cov}(X, Y)=0$?

(h) Choose the correct option for binomial distribution.

(i) Variance = Mean

(ii) Variance > Mean

(iii) Variance < Mean

2. Answer the following questions : 3×4=12

(a) A speaks the truth in 75% cases and B in 80%. In what percentages of cases are they likely to contradict each other while narrating the same incident?

(b) Define probability mass function and probability density function for a random variable X.

(c) The second moment about any point (a) is minimum when taken about the mean (μ), i.e.,

$$E(X-a)^2 \geq E(X-\mu)^2$$

Prove or disprove the above statement.

(d) State the three Poisson's postulates.

3. Answer any *two* parts from the following questions : 5×2=10

(a) What is meant by partition of a sample space S? If $H_i = (i = 1, 2, \cdots, n)$ is a partition of the sample space S, then for any event A, prove that

$$P(H_i / A) = \frac{P(H_i) P(A/H_i)}{\sum_{i=1}^{n} P(H_i) P(A/H_i)}$$

(b) Three machines A, B and C produce respectively 60%, 30% and 10% of the total number of items of a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C.

(c) A bag contains 5 balls and it is known how many of these are white. Two balls are drawn and are found to be white. What is the probability that all are white?

4. Answer any *two* parts from the following questions : 5×2=10

(a) Let F be the distribution function of a two-dimensional random variable (X, Y). If $a < b$ and $c < d$, then show that

$$P(a < X \le b, c < Y \le d) = F(b, d) + F(a, c) - F(a, d) - F(b, c)$$

(b) If X is a discrete random variable having probability mass function

Mass point	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$3k$	$4k$	k^2	$2k^2$	$7k^2 + k$

Determine (i) k, (ii) $P(X < 6)$ and (iii) $P(X \ge 6)$. 2+2+1=5

(c) The probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) = x + y, \quad 0 < x + y < 1$$
$$= 0, \quad \text{elsewhere}$$

Evaluate $P(X < \frac{1}{2}, Y > \frac{1}{4})$.

5. Answer any *two* parts from the following questions : 5×2=10

(a) From a bag containing 4 white and 6 red balls, three balls are drawn at random. Find the expected number of white balls drawn.

(b) For any two independent random variables X and Y, for which $E(X)$ and $E(Y)$ exist, show that

$$E(XY) = E(X)E(Y)$$

(c) The probability density function of a continuous bivariate distribution is given by the joint density function

$$f(x, y) = x + y, \quad 0 < x < 1, \; 0 < y < 1$$
$$= 0, \quad \text{elsewhere}$$

Find $E(X)$, $E(Y)$, $\text{var}(X)$, $\text{var}(Y)$ and $E(XY)$.

6. Answer any *two* parts from the following questions : 5×2=10

(a) If X is a Poisson distributed random variable with parameter μ, then show that $E(X) = \mu$ and $\text{var}(X) = \mu$.

(b) Show that normal distribution may be regarded as a limiting case of Poisson's distribution as the parameter $m \to \infty$.

(c) Define binomial distribution. What is the probability of guessing correctly at least six of the ten answers in a True-False objective test?

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