

2018

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatorics)

Full Marks : 60

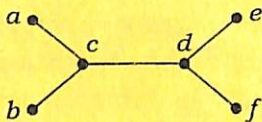
Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7
- (a) Write the multiplicative rule principle of combinatorics.
 - (b) In how many ways can five examinations be scheduled in a week so that no two examinations are scheduled on the same day, considering Sunday as a holiday?
 - (c) In how many ways can a committee of 5 persons be formed from 6 men and 4 women so as to include 3 men and 2 women?
 - (d) Draw a simple graph having four vertices each of degree 2.
 - (e) Draw complete graph K_4 .
 - (f) What is the length of a path?
 - (g) How many edges of a tree are having n vertices?

2. Answer the following questions : 2×4=8

- (a) A bag contains six white marbles and five red marbles. Find the number of ways that four marbles can be drawn from the bag if they must be the same colour.
- (b) How many vertices are there in a graph with 15 edges, if each vertex is of degree 3?
- (c) Show that there is only one path between every pair of vertices in a tree.
- (d) Find the radius and diameter of the tree shown below and show that diameter in a tree is not necessarily double of its radius :



3. Answer the following questions :

- (a) Give combination proof of the following identities : 2+3=5
- (i) $C(n, r) = C(n, n-r)$
- (ii) $C(n+1, r) = C(n, r) + C(n, r-1)$
- (b) There exists no simple graph corresponding the following degree sequences :

0, 2, 2, 3, 4

Justify the above statement.

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Or

Prove that for any graph G with six vertices, G or \bar{G} contains a triangle.

- (c) Let v be a point of a connected graph G . Then prove that the following statements are equivalent :

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- (i) v is a cut point of G .
- (ii) There exist points u and w distinct from v such that v is in every $u-w$ path.
- (iii) There exists a partition of the set of points $V - \{v\}$ into subsets U and W such that for any point $u \in U$ and $w \in W$, the point v is on every $u-w$ path.

Or

Let G be a connected graph with at least three points. If G is a block, then prove that every two points of G lie on a common cycle.

4. Answer any one part :

- (a) For any graph G , prove that

$$K(G) \leq \lambda(G) \leq \delta(G)$$

The symbols have their usual meaning.

Also show that the maximum vertex connectivity of a graph G with n vertices and e edges ($e > n - 1$) is the integral part of the number, $\frac{2e}{n}$.

$$7+3=10$$

- (b) State and prove Menger's theorem on graph. 10
5. Answer any *one* part :
- (a) Prove that a connected graph is Eulerian if and only if every vertex of G has even degree. 10
- (b) (i) If G is a simple graph with n vertices ($n \geq 3$) and if $\deg(v) + \deg(w) \geq n$ for every pair of non-adjacent vertices v and w , then prove that G is Hamiltonian. 7
- (ii) Under what conditions on r and s does the complete bipartite graph $K_{r,s}$ have a Hamiltonian circuit? 3
6. Answer any *one* part :
- (a) (i) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 7. 5
- (ii) Find the number of integral solutions of the equation $x + y + z = 18$ with the conditions that $x < 7$, $y < 8$ and $z < 9$. 5
- (b) (i) Find the number of non-negative solution of $x + y + z = 18$ with the conditions that $x \geq 3$, $y \geq 2$, $z \geq 1$. 5
- (ii) What is the probability that exactly one cell is empty if ten identical balls are distributed randomly into five distinct cells? 5
