

3 (Sem-3) STS M 2

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STATISTICS

(Major)

Paper : 3-2

(Distribution—I)

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions : 1×7=7

- (a) Under what condition, the binomial distribution possesses the additive property?
- (b) What is the relation between mean and variance of geometric distribution?
- (c) Give an example in which negative binomial distribution occurs.
- (d) Under what conditions hypergeometric distribution tends to binomial distribution?

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(Turn Over)

(2)

- (e) Write down the probability density function of Weibull distribution.
- (f) Name the distribution for which mean does not exist.
- (g) Under what condition mean and variance of exponential distribution are equal?

2. Answer the following questions : $2 \times 4 = 8$

- (a) If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively, find the probability that $X = Y$.
- (b) Derive moment generating function of geometric distribution.
- (c) If X is a random variable with a continuous distribution function F , then prove that $F(X)$ has a uniform distribution on $[0, 1]$.
- (d) If X follows standard Cauchy distribution, find a p.d.f. for X^2 and identify its distribution.

(3)

3. Answer any three of the following : $5 \times 3 = 15$

- (a) If X has negative binomial distribution with parameters (n, P) , prove that $M_X(t) = (Q - Pe^t)^{-n}$. Hence find m.g.f. of

$$z = \frac{X - nP}{\sqrt{nPQ}}$$

and deduce that z is asymptotically normal as $n \rightarrow \infty$.

- (b) Show that for a normal distribution with mean μ and variance σ^2 , the central moments satisfy the relation

$$\mu_{2n} = (2n - 1)\mu_{2n-2}\sigma^2$$

- (c) X_1 and X_2 are two independent random variables possessing same geometric distribution. Then show that

$$P(X_1 | X_1 + X_2 = n) = \frac{1}{n+1}; n = 0, 1, 2, \dots$$

- (d) If X and Y are independent gamma variates with parameters m and n respectively, show that the variables $U = X + Y$ and $V = \frac{X}{X + Y}$ are independently distributed and identify their distributions.

- (e) If X follows Laplace distribution with parameters λ and μ , find the r th moment about origin.

4. Answer any three of the following : $10 \times 3 = 30$

- (a) Let the random variable X be distributed uniformly on $(0,1)$ with p.d.f. $f(x) = 1, 0 \leq x \leq 1$. Assume that the conditional distribution of Y given $X = x$ has a binomial distribution with p.m.f.

$$P(Y = y | X = x) = \binom{n}{y} x^y (1-x)^{n-y};$$

$$y = 0, 1, 2, \dots, n$$

where n and x are the parameters of the distribution. Find—

- (i) the distribution of Y
- (ii) $E(Y)$
- (b) (i) Mention the chief characteristics of normal distribution. 5
- (ii) Let X be a one-parameter exponential distribution with parameter λ . Then for every constant a ($a > 0$), prove that $P(X \leq x+a | X \geq a) = P(X \leq x)$ for all $a > 0, x > 0$. 5
- (c) If X be a non-negative integer valued random variable satisfying
- $$P(X > i+j | X > i) = P(X \geq j)$$
- for any two positive integers i and j , then show that X must have a geometric distribution.

- (d) (i) Show that if the random variables X and Y are iid binomial variables with parameters (n, P) and (m, P) respectively, then the conditional distribution of $\frac{X}{X+Y}$ is hypergeometric. 4
- (ii) Prove that for negative binomial distribution
- $$P(x) = \binom{k+x-1}{x} q^x P^k, x = 0, 1, 2, \dots$$
- $$\mu_{r+1} = q \left(\frac{d\mu_r}{dq} + \frac{rk}{P^2} \mu_{r-1} \right) \quad 6$$
- (e) (i) Let $X \sim \beta_1(m, n)$ and $Y \sim \gamma(\lambda, m+n)$ be independent random variables, $(m, n, \lambda) > 0$. Find a p.d.f. for XY and identify the distribution. 5
- (ii) If X follows normal distribution with mean μ and standard deviation σ , find the distribution of $Y = a + bx$. 5
- (f) (i) Let X_i ($i = 1, 2, \dots, n$) is iid random variable having Weibull distribution. Show that the variable $Y = \min(X_1, X_2, \dots, X_n)$ also has Weibull distribution. 4

(6)

(ii) If X has a Poisson distribution with parameter λ , show that the distribution function of X is given by

$$F(x) = \frac{1}{\Gamma(x+1)} \int_{\lambda}^{\infty} e^{-t} t^x dt; x=0, 1, 2, \dots \quad 6$$
