2016

MATHEMATICS

(Major)

Paper : 5.6

(Optimization Theory)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Fill in the blanks : 1×7=7

(a) The set of all feasible solutions of an LPP is a _____ set.

(b) A system of m linear equations in n unknowns has at most _____ basic solutions.

(c) A BFS is a basic solution whose variables are _____.

(d) A BFS having more than $(n-m)$ variables 0, is named as _____, where the system consists of m linear equations in n unknowns.

(e) The objective function of an LPP is optimum at a _____ solution.

(f) If the jth primal variable x_j is unrestricted in sign, then in the dual problem, the jth constraint is an _____.

(g) In a balanced transportation problem with m sources and n destinations, the number of constraint equations is _____.

2. Answer the following questions : 2×4=8

(a) What do you mean by an LPP?

(b) The f.s. $x_1 = 1,\ x_2 = 0,\ x_3 = 1,\ z = 6$ to the system

$$x_1 + x_2 + x_3 = 2,$$
$$x_1 - x_2 + x_3 = 2,$$

$$\text{Min } Z = 2x_1 + 3x_2 + 4x_3,$$
$$x_i \geq 0 \quad i = 1, 2, 3$$

is not basic. Justify.

(c) Define convex set.

(d) Which of the points (0, 0), (0, 1), (1, 2), (1, 1), (4, 0) is an interior point of the convex hull of the above points?

3. Answer any *three* parts of the following : 5×3=15

(a) Upon completing the construction of his house, Mr. Somani discovered that 100 sq. ft. of plywood scrap and 80 sq. ft. of white pine scrap are in usable form for construction of tables and bookcases. It takes 16 sq. ft. of plywood and 2 sq. ft. of white pine to make a table and 12 sq. ft. of plywood and 16 sq. ft. of white pine for a bookcase. By selling the finished products to a local furniture store, Mr. Somani can realize a profit of ₹ 25 on each table and ₹ 20 on each bookcase. How may he most profitably use the left-over wood? Use graphical method to solve the problem.

(b) If S and T are any two convex sets in R^n, then for all scalars α, β, prove that the set $\alpha S + \beta T$ is also a convex set.

(c) If x is any f.s. to the primal

$$\text{Min } Z_p = \bar{C}\bar{x}$$

subject to $A\bar{x} \geq \bar{b}$, $\bar{x} \geq 0$

and w is any f.s. to the dual

$$\text{Max } Z_D = \bar{b}'w$$

subject to $A'\bar{w} \leq \bar{C}'$, $\bar{w} \geq 0$

then show that $Z_p \geq Z_D$.

(d) Give the dual of the following and solve :

$$\text{Min } Z = x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

4. Solve the following LPP by simplex method : 10

$$\text{Max } Z = 3x_1 + 5x_2 + 4x_3$$

subject to

$$2x_1 + 3x_2 \leq 8$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
$$2x_2 + 5x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Or

Find all BFS for the equations

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$ 10

5. Solve the following LPP by using the big M-method : 10

$$\text{Max } Z = x_1 + 2x_2 + 3x_3 - x_4$$

subject to

$$x_1 + x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Or

A diet-conscious housewife wishes to ensure certain minimum intake of vitamins A, B and C for the family. The minimum daily (quantity) needs of vitamins A, B and C for the family are respectively 30, 20 and 16 units. For the supply of these minimum vitamin requirements, the housewife relies

on two fresh foods. The first one provides 7, 5, 2 units of the three vitamins per gram respectively and the second one provides 2, 4, 8 units of the same three vitamins per gram of the foodstuff respectively. The first foodstuff costs ₹ 3 per gram and the second foodstuff costs ₹ 2 per gram.

(a) Formulate the underline LP problem.

(b) Write the dual problem.

(c) Solve the dual problem by using simplex method and also write the solution of the primal from the final simplex table of the dual. 4+2+4=10

6. Find a solution of the following transportation problem which will minimize the total cost : 10

$D \rightarrow$ $O \downarrow$	D_1	D_2	D_3	D_4	*Available* a_i
	1	2	1	4	30
	3	3	2	1	50
	4	2	5	9	20
Requirement b_j	20	40	30	10	

Or

A company is faced with the problem of assigning six different machines to five different jobs. The costs are estimated as follows (hundreds of rupees) :

Jobs $\rightarrow$ *Machines* $\downarrow$	J_1	J_2	J_3	J_4	J_5
M_1	2·5	5	1	6	1
M_2	2	5	1·5	7	3
M_3	3	6·5	2	8	3
M_4	3·5	7	2	9	4·5
M_5	4	7	3	9	6
M_6	6	9	5	10	6

Solve the problem assuming that the objective is to minimize the total cost. 10

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