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MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks
for the questions

1. Answer all the questions :   1×7=7

   (a) How many great circles can be drawn through two given points?

   (b) Define spherical triangle.

   (c) Explain what is meant by 'parallel of latitudes'.

   (d) At the position of transit of a star, what is the property of altitude and its zenith?

   (e) What is the relation between the linear velocity and perpendicular distance from the centre upon the tangent to the path of a central orbit?

(Turn Over)

(f) Stating the physical situation, define annular solar eclipse.

(g) State the position of the sun which is known as summer solstice.

2. Answer *all* the following questions : 2×4=8

(a) What are meant by ecliptic limits? Explain.

(b) Show that the sum of the three sides of a spherical triangle is less than the circumference of a great circle.

(c) Prove that for a right spherical triangle where $C = \pi/2$, $\cos A = \tan b \cot c$.

(d) If T is the orbital period of a planet, show that a small increment Δa in the semi-axis a will produce an increase $\dfrac{3T\Delta a}{2a}$ in the period.

3. Answer any *three* parts of the following : 5×3=15

(a) Explain about the dynamical significance of the Kepler's laws.

(b) Show that the velocity of a planet in its orbit has got two constant components, one perpendicular to the radius vector and the other perpendicular to the major axis.

(Continued)

(c) Distinguish between geocentric parallax and annual parallax of a star. Determine the effects of annual or stellar parallax on right ascension and declination.

(d) Explain with the help of neat diagrams the coordinate systems of celestial sphere.

(e) If a is the sun's altitude in the prime vertical at a place of latitude ϕ and L is its longitude, prove that

$$\phi = \sin^{-1}(\sin L \sin \varepsilon \operatorname{cosec} a)$$

4. In a spherical triangle, prove that

$$\cos a \cos C = \sin a \cot b - \sin C \cot B$$

Also prove that, if a be the side of an equilateral spherical triangle and a' that of its polar triangle, then

$$2\cos\frac{a}{2}\cos\frac{a'}{2} = 1 \qquad 6+4=10$$

5. Show that the mathematical condition for lunar eclipse to be possible of some kind is

$$\xi < D(1 - 2q\cos i + q^2)^{1/2} \operatorname{cosec} i$$

where $D = \alpha \pm \gamma_c$ for partial and total eclipse respectively,

$$q = \frac{\theta}{\phi} = \frac{\text{rate of increase of sun's longitude}}{\text{moon's angular velocity in its orbit}}$$

the other symbols carry their usual meanings. 10

(Turn Over)

6. Discuss the effects of refraction on sunrise and sunset.   10

Or

(a) Show that the retardation due to parallax in the time of rising of an object of geocentric parallax p seconds of arc and of declination δ is

$$\frac{1}{15}\frac{p}{\sqrt{(\cos^2\phi - \sin^2\delta)}} \text{ seconds,}$$

ϕ being the latitude of the place.   5

(b) If S is the semi-vertical angle of the tangent cone to the moon from the earth's centre when the moon's horizontal parallax is E and if S', P' be another similar parts, prove that the earth being supposed spherical

$$\frac{\sin S}{\sin S'} = \frac{\sin P}{\sin P'}$$

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