2016

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

(a) Write down the *n*th derivative of $\cos(2x+3)$.

(b) If $z = x^3 y^5 \phi(x/y)$, find the value of

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$$

(c) Find the expression for the subnormal to the curve $y^2 = 4ax$ at any point $P(x, y)$ on the curve.

(d) Write down the radius of curvature for the curve $s = c\tan\psi$.

(e) Write down the asymptotes to the curve $xy = a^2$.

(f) If $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right) + \cot^{-1}\left(\frac{y}{x}\right)$, $x \neq 0$, find $\frac{\partial f}{\partial x}$.

(g) If $I = \int \sqrt{x^2 - a^2}\, dx$, write down the expression for I.

(h) Write down the value of $\int_0^{\pi} |\cos x|\, dx$.

(i) What is the volume of the solid generated due to the revolution of the circle $x^2 + y^2 = a^2$ about x-axis?

(j) Evaluate $\int_{-\pi}^{\pi} |x| \sin x\, dx$.

2. Answer the following questions : 2×5=10

(a) If $y = \sin x \sin 2x \sin 3x$, find y_n.

(b) Show the pedal equation of the curve $r = e^{\theta}$ is $2p^2 = r^2$.

(c) Prove that $\int_0^{\pi} x \cos^4 x\, dx = \frac{3\pi^2}{16}$.

(d) Show that the perimeter of the circle $x^2 + y^2 = a^2$ is $2\pi a$.

(e) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

3. Answer the following : 5×4=20

(a) If $u = f(x, y)$ where $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) Trace the curve $x^3 + y^3 = 3axy$.

Or

Prove that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.

(c) Integrate $\int (\sqrt{\tan x} + \sqrt{\cot x})\, dx$.

Or

Integrate $\int \frac{dx}{(x^2 - 2x + 1)\sqrt{x^2 - 2x + 3}}$.

(d) Find the whole length of the loop of the curve $9y^2 = (x + 7)(x + 4)^2$.

4. Answer *either (a) or (b)* :

(a) *(i)* If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, $|x| < 1$, show that

$$(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$$ 6

(ii) If $y = x^{n-1} \log x$, show that $y_n = \frac{(n-1)!}{x}$. 4

(b) *(i)* If u is a homogeneous function of x and y of degree n, having continuous partial derivatives, prove that

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} \right)^2 u = n(n-1)u$$ 6

(ii) If $v = \sin^{-1} \frac{x^2 + y^2}{x+y}$, then show that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \tan v$$ 4

5. Answer *either (a) or (b)* :

(a) *(i)* Find the asymptotes of the curve $x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0$. 5

(ii) Show that for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

the radius of curvature at an extremity of the major axis is equal to half the latus rectum. 5

(b) Define cusp, isolated points, single cusp and double cusp. Find the position and nature of the multiple points on the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$. 4+6

6. *(a)* If $U_n = \int_0^{\pi/2} x^n \sin x \, dx \; (n \geq 1)$, show that

$$U_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)u_{n-2}$$ 5

(b) If $J_n = \int (a^2 + x^2)^{n/2} \, dx$, show that

$$J_n = \frac{x(a^2 + x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} J_{n-2}$$ 5

7. *(a)* Show that the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. 5

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